SWAN

In SWAN the waves are described with the two-dimensional wave action density spectrum, even when nonlinear phenomena dominate (e.g., in the surf zone). The rationale for using the spectrum in such highly nonlinear conditions is that, even in such conditions it seems possible to predict with reasonable accuracy this spectral distribution of the second order moment of the waves (although it may not be sufficient to fully describe the waves statistically). The spectrum that is considered in SWAN is the action density spectrum $N(\sigma,\theta)$ rather than the energy density spectrum $E(\sigma,\theta)$ since in the presence of currents, action density is conserved whereas energy density is not (e.g., Whitham, 1974). The independent variables are the relative frequency σ (as observed in a frame of reference moving with current velocity) and the wave direction (the direction normal to the wave crest of each spectral component). The action density is equal to the energy density divided by the relative frequency: $E(\sigma,\theta) = E(\sigma,\theta)/\sigma$. In SWAN this spectrum may vary in time and space.

Action balance equation In SWAN the evolution of the wave spectrum is described by the spectral action balance equation which for Cartesian coordinates is (e.g., Hasselmann et al., 1973):

$$\frac{\partial}{\partial t} N + \frac{\partial}{\partial x} c_x N + \frac{\partial}{\partial y} c_y N + \frac{\partial}{\partial \sigma} N + \frac{\partial}{\partial \theta} c_0 N = \frac{S}{\sigma} \qquad \text{(A1a)}$$

The first term in the left-hand side of this equation represents the local rate of change of action density in time, the second and third term represent propagation of action in geographical space (with propagation velocities c_x and c_y in *x*- and *y*- space, respectively). The fourth term represents shifting of the relative frequency due to variations in depths and currents (with propagation velocity c_σ in σ- space). The fifth term represents depth-induced and current-induced refraction (with propagation velocity c_σ in θ-space). The expressions for these propagation speeds are taken from linear wave theory (e.g., Whitham, 1974; Mei, 1983; Dingemans, 1997). The term S (= $S(\sigma,\theta)$) at the right hand side of the action balance equation is the source term in terms of energy density representing the effects of generation, dissipation and nonlinear wave-wave interactions. A brief summary of the formulations that are used for the various source terms in SWAN is given next. More details are given in Section 2.1.4.

In view of the use of SWAN at shelf sea or oceanic scale the user can choose to express the basic equation in spherical coordinates:

$$\frac{\partial}{\partial t} N + \frac{\partial}{\partial \lambda} c\lambda N + (\cos\varphi)^{1} \frac{\partial}{\partial \varphi} c_\varphi (\cos\varphi) N + \frac{\partial}{\partial \sigma} c_\sigma N + \frac{\partial}{\partial \theta} c_\theta N = \frac{S}{\sigma} \qquad \text{(A1b)}$$

with longitude, λ and latitude, φ.

Wind input

Transfer of wind energy to the waves is described in SWAN with a resonance mechanism (Phillips,1957) and a feed-back mechanism (Miles, 1957). The corresponding source term for these mechanisms is commonly described as the sum of linear and exponential growth:

$$S_{in}(\sigma, \theta) = A + BE(\sigma, \theta) \qquad \text{(A2)}$$

in which *A* and *B* depend on wave frequency and direction, and wind speed and direction. The effects of currents are accounted for in SWAN by using the apparent local wind speed and direction. The expression for the term *A* is due to Cavaleri and Malanotte-Rizzoli (1981) with a filter to avoid growth at frequencies lower than the Pierson-Moskowitz frequency (Tolman, 1992a). Two optional expressions for the coefficient *B* are used in the model. The first is taken from an early version of the WAM model (known as WAM Cycle 3, the WAMDI group, 1988). It is due to Snyder et al. (1981), rescaled in terms of friction velocity *U* by Komen et al. (1984). The drag coefficient to relate $U_{(}$ to the driving wind speed at 10 m elevation U_{10} is taken from Wu (1982). The second expression for *B* in SWAN is taken from the most recent version of the WAM model (known as WAM Cycle 4, Komen et al., 1994). It is due to Janssen (1991a) and it accounts explicitly for the interaction between the wind and the waves by considering atmospheric boundary layer effects and the roughness length of the sea surface. The corresponding set of equations is solved (as in the WAM model) with the iterative procedure of Mastenbroek et al. (1993).

Dissipation

The dissipation term of wave energy is represented by the summation of three different contributions:

whitecapping $S_{ds,w}(\sigma, \theta)$, bottom friction $S_{ds,b}(\sigma, \theta)$ and depth-induced breaking $S_{ds,br}(\sigma, \theta)$.
Whitecapping is primarily controlled by the steepness of the waves. In presently operating third-generation wave models (including SWAN) the whitecapping formulations are based on a pulse-based model (Hasselmann, 1974), as adapted by the WAMDI group (1988):

$$S_{ds,w}(\sigma,\theta) = -\Gamma \tilde{\sigma} \frac{k}{\tilde{k}} E(\sigma,\theta) \qquad \text{(A3)}$$

where Γ is a steepness dependent coefficient, k is wave number and $\tilde{\sigma}$ and $\tilde{K}$ denote a mean frequency and a mean wave number, respectively (cf. the WAMDI group, 1988). Komen et al. (1984) estimated the value of Γ by closing the energy balance of the waves in fully developed conditions. This implies that this value depends on the wind input formulation that is used. Since two expressions are used for the wind input in SWAN, also two values for Γ are used. The first is due to Komen et al. (1984, as in Cycle 3 of the WAM model). It is used in SWAN when the wind input coefficient *B* of Komen et al. (1984) is used. The second expression is an adaptation of this expression based on Janssen (1991a; as in Cycle 4 of the WAM model; see Janssen, 1991b; Günther et al., 1992). It is used when the wind input term *B* of Janssen (1991a) is used. Young and Banner (1992) and Banner and Young (1994) have shown that the results of closing the energy balance in this manner depend critically on the choice of a high-frequency cut-off frequency above which a diagnostic spectral tail is used. In SWAN this cut-off frequency is different from the one used in the WAM model. Differences in the growth rates between the WAM model and SWAN are therefore to be expected.

An alternative formulation for whitecapping is based on the Cumulative Steepness Method as described in Alkyon *et al.* (2002). With this method dissipation due to whitecapping depends on the steepness of the wave spectrum at and below a particular frequency. It is defined as (directionally dependent):

$$S_{st}(\sigma,\theta) = \int_0^{\sigma}\int_0^{2\pi} k^2 \left|\cos(\theta-\theta')\right|^m E(\sigma,\theta)\, d\sigma d\theta \qquad \text{(A4)}$$

In this expression the coefficient *m* controls the directional dependence. It is expected that this coefficient will be order 1 if the straining mechanism is dominant, *m* is more than 10 if other mechanism play a role (e.g. instability that occurs when vertical acceleration in the waves becomes greater than gravity). Default in SWAN is *m* = 2.

The new whitecapping source term is given by

$$S_{wc}^{st}(\sigma,\theta) = -C_{wc}^{st} S_{st}(\sigma,\theta) E(\sigma,\theta) \qquad \text{(A5)}$$

with C_{wc}^{st} a tuneable coefficient.

Depth-induced dissipation may be caused by bottom friction, by bottom motion, by percolation or by back-scattering on bottom irregularities (Shemdin et al., 1978). For continental shelf seas with sandy bottoms, the dominant mechanism appears to be bottom friction (e.g., Bertotti and Cavaleri, 1994) which can generally be represented as:

$$S_{ds,b}(\sigma,\theta) = -C_{bottom} \frac{\sigma^2}{g^2 \sinh^2(kd)} E(\sigma,\theta) \qquad \text{(A6)}$$

in which C_{bottom} is a bottom friction coefficient. A large number of models has been proposed since the pioneering paper of Putnam and Johnson (1949). Hasselmann et al. (JONSWAP, 1973) suggested to use an empirically obtained constant. It seems to perform well in many different conditions as long as a suitable value is chosen (typically different for swell and wind sea; Bouws and Komen, 1983). A nonlinear formulation based on drag has been proposed by Hasselmann and Collins (1968) which was later simplified by Collins (1972). More complicated, eddy viscosity models have been developed by Madsen et al. (1988; see Weber, 1991a) and by Weber (1989, 1991a, 1991b). Considering the large variations in bottom conditions in coastal areas (bottom material, bottom roughness length, ripple height etc.), there is no field data evidence to give preference to a particular friction model (Luo and Monbaliu, 1994). For this reason, the simplest of each of

these types of friction models has been implemented in SWAN: the empirical JONSWAP model of Hasselmann et al. (1973), the drag law model of Collins (1972) and the eddy-viscosity model of Madsen et al. (1988). The effect of a mean current on the wave energy dissipation due to bottom friction is not taken into account in SWAN. The reasons for this are given by Tolman (1992b) who argues that state-of-the-art expressions vary too widely in their effects to be acceptable. He found that the error in finding a correct estimate of the bottom roughness length scale has a much larger impact on the energy dissipation rate than the effect of a mean current.

The process of depth-induced wave-breaking is still poorly understood and little is known about its spectral modelling. In contrast to this, the total dissipation (i.e., integrated over the spectrum) due to this type of wave breaking can be well modelled with the dissipation of a bore applied to the breaking waves in a random field (Battjes and Janssen, 1978; Thornton and Guza, 1983). Laboratory observations (e.g., Battjes and Beji, 1992; Vincent et al. 1994; Arcilla et al., 1994 and Eldeberky and Battjes, 1996) show that the shape of initially uni-modal spectra propagating across simple (barred) beach profiles, is fairly insensitive to depth-induced breaking. This has led Eldeberky and Battjes (1995) to formulate a spectral version of the bore model of Battjes and Janssen (1978) which conserves the spectral shape. Expanding their expression to include directions, the expression that is used in SWAN is:

$$S_{ds,br}(\sigma,\theta) = \frac{D_{tot}}{E_{tot}} E(\sigma,\theta) \qquad \text{(A7)}$$

in which E_{tot} is the total wave energy and D_{tot} <0 is the rate of dissipation of the total energy due to wave breaking according to Battjes and Janssen (1978). Adding a quadratic dependency on frequency as suggested by Mase and Kirby (1992, supported by Elgar et al., 1997) seems to have no noticeable effect on the SWAN results. Chen and Guza (1997) inferred from observations and simulations with a Boussinesq model that the high-frequency levels are insensitive to such frequency dependency because an increased dissipation at high frequencies is compensated approximately by increased nonlinear energy transfer (but they did find the frequency dependency to be relevant in time domain). The value of D_{tot} depends critically on the breaking parameter $\gamma = H_{max} / d$ (in which H_{max} is the maximum possible individual wave height in the local water depth *d*). In SWAN a constant value and a variable value are available. The constant value is $\gamma = 0.73$ (the mean value of the data set of Battjes and Stive, 1985).

Nonlinear wave-wave interactions

In deep water, quadruplet wave-wave interactions dominate the evolution of the spectrum. They transfer wave energy from the spectral peak to lower frequencies (thus moving the peak frequency to lower values) and to higher frequencies (where the energy is dissipated by whitecapping). In very shallow water, triad wave-wave interactions transfer energy from lower frequencies to higher frequencies often resulting in higher harmonics (Beji and Battjes, 1993; low-frequency energy generation by triad wave-wave interactions is not considered here).

A full computation of the quadruplet wave-wave interactions is extremely time consuming and not convenient in any operational wave model. Nevertheless, the current version of SWAN (40.41) has two options to compute the Boltzmann integral in an exact manner. The first approach is the so-called FD-RIAM technique as proposed by Hashimoto *et al.* (1998). This approach enables to capture the frequency shift and the spectral shape changes as water depth decreases. The FD-RIAM method as implemented in SWAN has been successfully applied to the shallow water Lake Georgia in Australia; see Hashimoto *et al.* (2003). The second approach is the exact method developed by Webb, Tracy and Resio (WRT) (Resio *et al*, 2001). This algorithm was reprogrammed by Van Vledder, bearing the name Xnl (Van Vledder and Bottema, 2003). This method is also enable to capture the frequency shift and the spectral shape changes as water depth decreases.

A number of techniques, based on parametric methods or other types of approximations have been proposed to improve computational speed (see Young and Van Vledder (1993) for a review). In SWAN, the computations are carried out with the Discrete Interaction Approximation (DIA) of Hasselmann et al. (1985). This DIA has been found to be quite successful in describing the essential features of a developing wave spectrum; see Komen et al. (1994). For uni-directional waves, this approximation is not valid. In fact, the quadruplet interaction coefficient for these waves is nearly zero (G.Ph. van Vledder, personal communication, 1996). For finite-depth applications, Hasselmann and Hasselmann (1981) have shown that for a JONSWAP-type spectrum the quadruplet wave-wave interactions can be scaled with a simple expression.

In some cases, the DIA technique may not be accurate enough. In Hashimoto *et al.* (2003), it was demonstrated that the accuracy of the DIA may be improved by increasing the number of quadruplet configurations. They proposed a Multiple DIA with up to 6 wave number configurations.

A first attempt to describe triad wave-wave interactions in terms of a spectral energy source term was made by Abreu et al. (1992). However, their expression is restricted to non-dispersive shallow water waves and is therefore not suitable in many practical applications of wind waves. The breakthrough in the development came with the work of Eldeberky and Battjes (1995) who transformed the amplitude part of the Boussinesq model of Madsen and Sørensen (1993) into an energy density formulation and who parameterized the biphase of the waves on the basis of laboratory observations (Battjes and Beji, 1992; Arcilla et al., 1994). A discrete triad approximation (DTA) for co-linear waves was subsequently obtained by considering only the dominant self-self interactions. Their model has been verified with flume observations of long-crested, random waves breaking over a submerged bar (Beji and Battjes, 1993) and over a barred beach (Arcilla et al., 1994). The model appeared to be fairly successful in describing the essential features of the energy transfer from the primary peak of the spectrum to the super harmonics. A slightly different version, the Lumped Triad Approximation (LTA) was later derived by Eldeberky (1996). This LTA technique is used in SWAN.

Obstacles

SWAN can estimate wave transmission through a (line-)structure such as a breakwater (dam). Such an obstacle will affect the wave field in two ways, first it will reduce the wave height locally all along its length, and second it will cause diffraction around its end(s). The model is not able to account for diffraction. In irregular, short-crested wave fields, however, it seems that the effect of diffraction is small, except in a region less than one or two wavelengths away from the tip of the obstacle (Booij et al., 1993). Therefore the model can reasonably account for waves around an obstacle if the directional spectrum of incoming waves is not too narrow. Since obstacles usually have a transversal area that is too small to be resolved by the bottom grid in SWAN, an obstacle is modelled as a line. If the crest of the breakwater is at a level where (at least part of the) waves can pass over, the transmission coefficient K_t (defined as the ratio of the (significant) wave height at the downwave side *t* of the dam over the (significant) wave height at the upwave side) is a function of wave height and the difference in crest level and water level. The expression is taken from Goda et al. (1967):

$$K_t = 0.5\left[1-\sin\left(\frac{\pi}{2\alpha}\left(\frac{F}{H_i}+\beta\right)\right)\right] \quad \text{for} \quad -\beta-\alpha<\frac{F}{H_i}<\alpha-\beta \tag{A8}$$

where F = h - d is the freeboard of the dam and where H_i is the incident (significant) wave height at the upwave side of the obstacle (dam), h is the crest level of the dam above the reference level (same as reference level of the bottom), d the mean water level relative to the reference level, and the coefficients α, β depend on the shape of the dam (Seelig, 1979):

case	α	β
vertical thin wall	1.8	0.1
caisson	2.2	0.4
dam with slope 1:3/2	2.6	0.15

The above expression is based on experiments in a wave flume, so strictly speaking it is only valid for normal incidence waves. Since there are no data available on oblique waves it is assumed that the transmission coefficient does not depend on direction. Another phenomenon that is to be expected is a change in wave frequency since often the process above the dam is highly nonlinear. Again there is little information available, so in SWAN it is assumed that the frequencies remain unchanged over an obstacle (only the energy scale of the spectrum is affected and not the spectral shape).

First-, second-and third-generation mode

SWAN can operate in first-, second-and third-generation mode. The first-and second-generation mode are essentially those of Holthuijsen and de Boer (1988) (first-generation with a constant Phillips constant of 0.0081; second-generation with a variable Phillips constant). An overview of the options is given in the Table below.

	generation mode of SWAN		
	1st	2nd	3rd

Linear wind growth:	Cavaleri & Malanotte-Rizzoli (1981) [modified]	x	x	
	Cavaleri & Malanotte-Rizzoli (1981)			x
Exponential wind growth:	Snyder et al. (1981) [modified]	x	x	
	Snyder et al. (1981)			x1
	Janssen (1989, 1991)			x2
Whitecapping:	Holthuijsen and de Boer (1988)	x3	x4	
	Komen et al. (1984)			x1
	Janssen (1991), Komen et al. (1994)			x2
Quadruplet interaction:	Hasselmann et al. (1985)			x
Triad interactions:	Eldeberky (1996)	x	x	x
Depth-induced breaking:	Battjes & Janssen (1978)	x	x	x
Bottom friction:	Hasselmann et al. JONSWAP (1973)	x	x	x
	Collins (1972)	x	x	x
	Madsen et al. (1988)	x	x	x
Obstacle transmission:	Seelig (1979)	x	x	x

For SWAN running in a third generation mode, the following combinations of the input and whitecapping parameterizations are used (indicated with [1] and [2], see command GEN3):

[1] gives the wind input and whitecapping formulations as used in WAM Cycle 3.

[2] gives the wind input and whitecapping formulations as used in WAM Cycle 4.

[3] Pierson-Moskowitz spectrum as upper limit.

[4] scaled Pierson-Moskowitz spectrum as upper limit.

Wave-induced set-up

In a 1D case, the computation of the wave-induced set-up is based on the vertically integrated momentum balance equation which reduces to a balance between the gradient of the wave radiation stress and the hydrostatic pressure gradient (no wave-induced currents exist). In a 2D case, the computation of the wave-induced set-up is based on the divergence of the vertically integrated momentum balance equation being zero.

Diffraction

To accommodate diffraction in SWAN simulations, a phase-decoupled refraction-diffraction approximation is suggested (Holthuijsen *et al*, 2003). It is expressed in terms of the directional turning rate of the individual wave components in the 2D wave spectrum. The approximation is based on the mild-slope equation for refraction and diffraction, omitting phase information. It does therefore not permit coherent wave fields in the computational domain.

Detailed formulation

The complete expressions for the physical processes of generation, dissipation and nonlinear wave-wave interactions that are available in the SWAN model are given here.

Input by wind (S_{in})
Wave growth by wind is described by:

$$S_{in}(\sigma,\theta) = A + B E(\sigma,\theta)$$

(B1)

in which A describes linear growth and BE exponential growth. It should be noted that the SWAN model is driven by the wind speed at 10 m elevation U_{10} whereas the computations use the friction velocity U_*. For the WAM Cycle 3 formulation the transformation from U_{10} to U_* is obtained with

$$U_*^2 = C_D U_{10}^2 \tag{B2}$$

in which C_D is the drag coefficient from Wu (1982):

$$C_D(U_{10}) = \begin{cases} 1.2875\times10^{-3} & \text{for} \quad U_{10} < 7.5\ \text{m/s} \\ (0.8 + 0.065\ \text{s/m} \times U_{10})\times10^{-3} & \text{for} \quad U_{10} \geq 7.5\ \text{m/s} \end{cases} \tag{B3}$$

For the WAM Cycle 4 formulations, the computation of U_* is an integral part of the source term.

Linear growth by wind

For the linear growth term A, the expression due to Cavaleri and Malanotte-Rizzoli (1981) is used with a filter to eliminate wave growth at frequencies lower than the Pierson-Moskowitz frequency (Tolman, 1992a) (note that in his equation (10) the power of 10^{-5} should be 10^{-3}; H. Tolman, personal communication, 1995):

$$A = \frac{1,5\times10^{-3}}{g^2\,2\pi}\left[U_* \max\left[0\ ,\ \cos(\theta-\theta_w)\right]\right]^4 H \quad , \tag{B4}$$

$$H = \exp\left(-(\sigma/\sigma_{PM}^*)^{-4}\right) \quad \text{with} \quad \sigma_{PM}^* = \frac{0.13g}{28\,U_*}2\pi \quad ,$$

in which θ_w is the wind direction, H is the filter and $\sigma_{PM}*$ is the peak frequency of the fully developed sea state according to Pierson and Moskowitz (1964; reformulated in terms of friction velocity).

Exponential growth by wind:

Two expressions for exponential growth by wind are optionally available in the SWAN model. The first expression is due to Komen et al. (1984). Their expression is a function of $\frac{U_*}{c_{ph}}$:

$$B = \max\left[0\ ,\ 0.25\,\frac{\rho_a}{\rho_w}\left[28\,\frac{U_*}{c_{ph}}\cos(\theta-\theta_w) - 1\right]\right]\sigma \quad , \tag{B5}$$

in which c_{ph} is the phase speed and ρ_a and ρ_w are the density of air and water, respectively. This expression is also used in WAM Cycle 3 (the WAMDI group, 1988). The second expression is due to Janssen (1989,1991). It is based on a quasi-linear wind-wave theory and is given by:

$$B = \beta\,\frac{\rho_a}{\rho_w}\left(\frac{U_*}{c_{ph}}\right)^2 \max\left[0\ ,\ \cos(\theta-\theta_w)\right]^2 \sigma \quad , \tag{B6}$$

where β is the Miles "constant". In the theory of Janssen (1991), this Miles "constant" is estimated from the non-dimensional critical height λ:

$$\begin{cases} \beta = \dfrac{1.2}{\kappa^2}\,\lambda \ln^4\lambda \quad , & \lambda \le 1 \\ \lambda = \dfrac{g z_e}{c_{ph}^2}\, e^{r} \quad , & r = \kappa c / \left| U_* \cos(\theta - \theta_w) \right| \end{cases} \quad , \tag{B7}$$

where k is the Von Karman constant, equal to 0.41 and z_e is the effective surface roughness. If the non-dimensional critical height λ>1, the Miles constant β is set equal 0. Janssen (1991) assumes that the wind profile is given by:

$$U(z) = \frac{U_*}{\kappa} \ln\left(\frac{z + z_e - z_o}{z_e} \right) \tag{B8}$$

in which $U(z)$ is the wind speed at height z (10 m in the SWAN model) above the mean water level, zo is the roughness length. The effective roughness length z_e depends on the roughness length zo and the sea state through the wave induced stress T_w and the total surface stress T:

$$z_e = \frac{z_o}{\sqrt{1 - \tau_w/\tau}} \qquad \text{and} \qquad z_o = \hat{\alpha}\,\frac{U_*^2}{g} \quad . \tag{B9}$$

The second of these two equations is a Charnock-like relation in which $\hat{\alpha}$ is a constant equal to 0.01. The wave stress $\underline{T}_w$ vector is given by:

$$\underline{\tau}_w = \rho_w \int_0^{2\pi}\int_0^{\infty} \sigma\, B\, E(\sigma,\theta)\, \frac{\underline{k}}{k}\, d\sigma\, d\theta \quad . \tag{B10}$$

The value of U_* can be determined for a given wind speed U_{10} and a given wave spectrum $E(\sigma, \theta)$ from the above set of equations. In the SWAN model the iterative procedure of Mastenbroek et al. (1993) is used. This set of expressions ((B-6) through (B-10)) is also used in WAM Cycle 4 (Komen et al., 1994).

Dissipation of wave energy (S_{ds})

Whitecapping

The processes of whitecapping in the SWAN model is represented by the pulse-based model of Hasselmann (1974). Reformulated in terms of wave number (rather than frequency) so as to be applicable in finite water depth (cf. the WAMDI group, 1988), this expression is:

$$S_{ds,w}(\sigma,\theta) = -\Gamma\,\tilde{\sigma}\,\frac{k}{\tilde{k}}\,E(\sigma,\theta), \tag{B11}$$

where $\tilde{\sigma}$ and $\tilde{k}$ denote the mean frequency and the mean wave number (for expressions see below) respectively and the coefficient Γ depends on the overall wave steepness. This steepness dependent coefficient, as given by the WAMDI group (1988), has been adapted by Günther et al. (1992) based on Janssen (1991a, see Janssen, 1991b):

$$\Gamma = \Gamma_{KJ} = C_{ds}\left((1-\delta) + \delta \frac{k}{\tilde{k}} \right) \left(\frac{\tilde{s}}{\tilde{s}_{PM}} \right)^{p} \quad , \tag{B12}$$

For $\delta = 0$ the expression of Γ reduces to the expression as used by the WAMDI group (1988). The coefficients C_{ds}, δ and m are tunable coefficients, $\tilde{s}$ is the overall wave steepness (defined below), $\tilde{s}_{PM}$ is the value of $\tilde{s}$ for the Pierson-Moskowitz spectrum (1964; $\tilde{s}_{PM} = (3.02\times10^{-3})^{½}$). This overall wave steepness $\tilde{s}$ is defined as:

$$\tilde{s} = \tilde{k} \sqrt{E_{tot}} \quad . \tag{B13}$$

The mean frequency $\tilde{\sigma}$, the mean wave number $\tilde{k}$ and the total wave energy: E_{tot} are defined as (cf. the WAMDI group, 1988):

$$\tilde{\sigma} = \left(E_{tot}^{-1} \int_0^{2\pi} \int_0^{\infty} \frac{1}{\sigma} E(\sigma,\theta)\, d\sigma\, d\theta \right)^{-1} \quad , \tag{B14}$$

$$\tilde{k} = \left(E_{tot}^{-1} \int_0^{2\pi} \int_0^{\infty} \frac{1}{\sqrt{k}} E(\sigma,\theta)\, d\sigma\, d\theta \right)^{-2} \quad ,$$

$$E_{tot} = \int_0^{2\pi} \int_0^{\infty} E(\sigma,\theta)\, d\sigma\, d\theta \quad . \tag{B15}$$

The values of the tunable coefficients *Cds* and δ and exponent ρ in this model have been obtained by Komen et al. (1984) and Janssen (1992) by closing the energy balance of the waves in idealized wave growth conditions (both for growing and fully developed wind seas) for deep water. This implies that coefficients in the steepness dependent coefficient depend on the wind input formulation that is used. Since two different wind input formulations are used in the SWAN model, two sets of coefficients are used. For the wind input of Komen et al. (1984; corresponding to WAM Cycle 3; the WAMDI group, 1988):
Cds = 2.36×10^{-5}, $\delta = 0$ and $\rho = 4$. Janssen (1992) and also Günther (1992) obtained (assuming $p = 4$) $C_{ds} = 4.10\times10^{-5}$ and $\delta = 0.5$ (as used in the WAM cycle 4; Komen et al., 1994).

Bottom friction
The bottom friction models that have been selected for SWAN are the empirical model of JONSWAP (Hasselmann et al., 1973), the drag law model of Collins (1972) and the eddy-viscosity model of Madsen et al. (1988). The formulations for these bottom friction models can all be expressed in the following form:

$$S_{ds,b}(\sigma,\theta) = - C_{bottom} \frac{\sigma^2}{g^2 \sinh^2(kd)} E(\sigma,\theta) \quad , \tag{B16}$$

in which C_{bottom} is a bottom friction coefficient that generally depends on the bottom orbital motion represented by U_{rms}:

$$U_{rms}^2 = \int_0^{2\pi} \int_0^{\infty} \frac{\sigma^2}{\sinh^2(kd)} E(\sigma,\theta)\, d\sigma\, d\theta \quad . \tag{B17}$$

Hasselmann et al. (1973) found from the results of the JONSWAP experiment $C_{bottom}=C_{JON}=0.038$ m^2 s^{-3} for swell conditions. Bouws and Komen (1983) selected a bottom friction coefficient of $C_{JON}=0.067$ m^{2}s^{-3} for fully developed wave conditions in shallow water. Both values are available in SWAN.

The expression of Collins (1972) is based on a conventional formulation for periodic waves with the appropriate parameters adapted to suit a random wave field. The dissipation rate is calculated with the conventional bottom friction formulation of Eq. B1 in which the bottom friction coefficient is $C_{bottom}=C_f\, gU_{rms}$ with C_f=0.015 (Collins, 1972). (Note that Collins (1972) contains an error in the expression due to an erroneous Jacobean transformation; see page A-16 of Tolman, 1990).

Madsen et al. (1988) derived a formulation similar to that of Hasselmann and Collins (1968) but in their model the bottom friction factor is a function of the bottom roughness height and the actual wave conditions. Their bottom friction coefficient is given by:

$$C_{bottom} = f_w \frac{g}{\sqrt{2}} U_{rms} \quad , \tag{B18}$$

in which f_w is a non-dimensional friction factor estimated by using the formulation of Jonsson (1966; cf. Madsen et al., 1988):

$$\frac{1}{4\sqrt{f_w}} + \log_{10}\left[\frac{1}{4\sqrt{f_w}}\right] = m_f + \log_{10}\left[\frac{a_b}{K_N}\right] \quad , \tag{B19}$$

in which m_f=-0.08 (Jonsson and Carlsen, 1976) and a_b is a representative near-bottom excursion amplitude:

$$a_b^2 = 2\int_0^{2\pi}\int_0^{\infty} \frac{1}{\sinh^2(kd)} E(\sigma,\theta)\, d\sigma\, d\theta \quad . \tag{B20}$$

and K_N is the bottom roughness length scale. For values of a_b / K_N smaller than 1.57 the friction factor f_w is 0.30 (Jonsson, 1980).

Depth-induced wave breaking

To model the energy dissipation in random waves due to depth-induced breaking, the bore-based model of Battjes and Janssen (1978) is used in SWAN. The mean rate of energy dissipation per unit horizontal area due to wave breaking D_{tot} is expressed as:

$$D_{tot} = -\frac{1}{4}\alpha_{BJ} Q_b \left(\frac{\bar{\sigma}}{2\pi}\right) H_m^2 \quad , \tag{B21}$$

in which σ_{BJ}=1 in SWAN, Q_b is the fraction of breaking waves determined by:

$$\frac{1-Q_b}{\ln Q_b} = -8\frac{E_{tot}}{H_m^2} \quad , \tag{B22}$$

in which H_m is the maximum wave height that can exist at the given depth and $\bar{\sigma}$ is a mean frequency defined as:

$$\bar{\sigma} = E_{tot}^{-1}\int_0^{2\pi}\int_0^{\infty} \sigma\, E(\sigma,\theta)\, d\sigma\, d\theta \quad .$$

(B23)

The fraction of depth-induced breakers (Q_b) is determined in SWAN with

$$
\begin{cases}
Q_b = 0 & \text{for } \beta \le 0.2 \\
Q_b = Q_0 - \beta^2 \dfrac{Q_0 - \exp((Q_0 - 1)/\beta^2)}{\beta^2 - \exp((Q_0 - 1)/\beta^2)} & \text{for } 0.2 < \beta < 1 \\
\qquad \text{one step Newton-Raphson iteration)} & \\
Q_b = 1 & \text{for } \beta \ge 1
\end{cases}
\tag{B24}
$$

where $\beta = H_{rms} / H_{\max}$. For $\beta \le 0.5$, $Q_0 = 1$ and for $0.5 < \beta \le 1$, $Q_0 = (2\beta - 1)^2$.

Extending the expression of Eldeberky and Battjes (1995) to include the spectral directions, the dissipation for a spectral component per unit time is calculated in SWAN with:

$$S_{ds,br}(\sigma,\theta) = D_{tot}\frac{E(\sigma,\theta)}{E_{tot}}. \tag{B25}$$

The maximum wave height H_m is determined in SWAN with $H_m = \gamma d$, in which γ is the breaker parameter and d is the total water depth (including the wave-induced set-up if computed by SWAN). In the literature, this breaker parameter γ is often a constant or it is expressed as a function of bottom slope or incident wave steepness (see e.g., Galvin, 1972; Battjes and Janssen, 1978; Battjes and Stive, 1985; Arcilla and Lemos, 1990; Kaminsky and Kraus, 1993; Nelson, 1987, 1994). Since SWAN is locally defined, the dependency on incident wave steepness cannot be used. Instead, the other two options (constant value or bottom-slope dependent) were used in SWAN version 40.01 and older to determine the value of the breaker parameter. Since SWAN version 40.11, the option of Nelson has been removed as the results of SWAN were better with the option of a constant value.

In the publication of Battjes and Janssen (1978) in which the dissipation model is described, a constant breaker parameter, based on Miche's criterion, of γ=0.8 was used. Battjes and Stive (1985) re-analyzed wave data of a number of laboratory and field experiments and found values for the breaker parameter varying between 0.6 and 0.83 for different types of bathymetry (plane, bar-trough and bar) with an average of 0.73. From a compilation of a large number of experiments Kaminsky and Kraus (1993) have found breaker parameters in the range of 0.6 to 1.59 with an average of 0.79.

Nonlinear wave-wave interactions (*Snl*)

Quadruplet wave-wave interactions

The quadruplet wave-wave interactions are computed with the Discrete Interaction Approximation (DIA) as proposed by Hasselmann et al. (1985). Their source code (slightly adapted by Tolman, personal communication, 1993) has been used in the SWAN model. In the Discrete Interaction Approximation two quadruplets of wave numbers are considered, both with frequencies:

$$
\begin{aligned}
\sigma_1 &= \sigma_2 = \sigma \\
\sigma_3 &= \sigma(1+\lambda) = \sigma^+ \\
\sigma_4 &= \sigma(1-\lambda) = \sigma^-
\end{aligned}
\quad , \tag{B26}
$$

where λ is a constant coefficient set equal to 0.25. To satisfy the resonance conditions for the first quadruplet, the wave number vectors with frequency σ_3 and σ_4 lie at an angle of θ_1=-11.5^o and θ_2=33.6^o to the two identical wave number vectors with frequencies σ_1 and σ_2. The second quadruplet is the mirror of this first quadruplet (the wave number vectors with frequency σ_3 and σ_4 lie at mirror angles of θ_3=11.5^o and θ_4=

-33.6°).

Within this discrete interaction approximation, the source term $S_{nl4}(\sigma, \theta)$ is given by:

$$S_{nl4}(\sigma,\theta) = S^{*}_{nl4}(\sigma,\theta) + S^{**}_{nl4}(\sigma,\theta) \quad , \tag{B27}$$

where S^{*}_{nl4} refers to the first quadruplet and *Snl4* to the second quadruplet (he expressions for S^{**}_{nl4} are identical to those for S^{*}_{nl4} for the mirror directions) and:

$$S^{*}_{nl4}(\sigma,\theta) = 2\delta S_{nl4}(\alpha_1\sigma,\theta) - \delta S_{nl4}(\alpha_2\sigma,\theta) - \delta S_{nl4}(\alpha_3\sigma,\theta) \quad , \tag{B28}$$

in which α_1=1, α_2=(1+λ) and α_3=(1-λ). Each of the contributions (i=1,2,3) is:

$$\left(\frac{\sigma}{2\pi}\right)^{11}\left[E^2(\alpha_i\sigma,\theta)\left(\frac{E(\alpha_i\sigma^{+},\theta)}{(1+\lambda)^4} + \frac{E(\alpha_i\sigma^{-},\theta)}{(1-\lambda)^4}\right) 2\,\frac{E(\alpha_i\sigma,\theta)\,E(\alpha_i\sigma^{+},\theta)\,E(\alpha_i\sigma^{-},\theta)}{(1-\lambda^2)^4}\right] \tag{B29}$$

The constant $C_{nl4} = 3\times10^{7}$. Following Hasselmann and Hasselmann (1981), the quadruplet interaction in finite water depth is taken identical to the quadruplet transfer in deep water multiplied with a scaling factor R:

$$S_{nl4,finitedepth} = R(k_p d)\, S_{nl4,infinitedepth} \, , \tag{B30}$$

where R is given by:

$$R(k_p d) = 1 + \frac{C_{sh1}}{k_p d}\left(1 - C_{sh2}\cdot k_p d\right)\exp\left(C_{sh3}\cdot k_p d\right) \quad , \tag{B31}$$

in which k_p is the peak wave number of the JONSWAP spectrum for which the original computations were carried out. The values of the coefficients are: C_{sh1} = 5.5, C_{sh2} = 6/7 and C_{sh3} = -1.25. In the shallow water limit, i.e., *kpd* 60 the nonlinear transfer tends to infinity. Therefore a lower limit of $k_p d$=0.5 is applied (cf. WAM Cycle 4; Komen et al., 1994), resulting in a maximum value of $R\ (k_p d)$=4.43. To increase the model robustness in case of arbitrarily shaped spectra, the peak wave number k_p is replaced by k_p = 0.75 $\tilde{K}$ (cf. Komen et al., 1994).

In Hashimoto *et al.* (2003), it was demonstrated that the accuracy of the DIA may be improved by increasing the number of quadruplet configurations. They proposed a Multiple DIA with up to 6 wave number configurations with different values of λ and C_{nl4}. These have been optimized by means of an exact method (RIAM). These optimum values are default when choosing MDIA in SWAN (see command GEN3).

Triad wave-wave interactions

The Lumped Triad Approximation (LTA) of Eldeberky (1996), which is a slightly adapted version of the Discrete Triad Approximation of Eldeberky and Battjes (1995) is used in SWAN in each spectral direction:

$$S_{nl3}(\sigma,\theta) = S^{-}_{nl3}(\sigma,\theta) + S^{+}_{nl3}(\sigma,\theta) \tag{B32}$$

with

$$S^{+}_{nl3}(\sigma,\theta) = \max\left\{0, \alpha_{EB}\,2\pi\, c\, c_g J^2 \left|\sin(\beta)\right| \left\{E^2(\sigma/2,\theta) - 2E(\sigma/2,\theta)\,E(\sigma,\theta)\right\}\right\} \tag{B33}$$

and

$$S_{nl3}^{-}(\sigma,\theta) = -2\,S_{nl3}^{+}(2\sigma,\theta) \tag{B34}$$

in which α_{EB} is a tunable proportionality coefficient. The biphase β is approximated with

$$\beta = -\frac{\pi}{2} + \frac{\pi}{2}\tanh\left(\frac{0.2}{Ur}\right) \tag{B35}$$

with Ursell number *Ur*

$$Ur = \frac{g}{8\sqrt{2}\,\pi^2}\,\frac{H_s\,\bar{T}^2}{d^2} \tag{B36}$$

with $\bar{T}$ = 2 n/$\bar{\sigma}$. Usually, the triad wave-wave interactions are calculated only for 0.1 ≤ *Ur* ≤ 10. But for stability reasons, it is calculated for the whole range 0 ≤ *Ur* ≤ 10. This means that both quadruplets and triads are computed at the same time. The interaction coefficient *J* is taken from Madsen and Sørensen (1993):

$$J = \frac{k_{\sigma/2}^2\,(g\,d + 2\,c_{\sigma/2}^2)}{k_\sigma\,d\,(g\,d + \frac{2}{15}\,g\,d^3\,k_\sigma^2 - \frac{2}{5}\,\sigma^2\,d^2)} \tag{B37}$$

Wave-induced set-up

In a geographic 1D case the computation of the wave induced set-up is based on the vertically integrated momentum balance equation which is a balance between the wave force (gradient of the wave radiation stress normal to the coast) and the hydrostatic pressure gradient (note that the component parallel to the coast causes wave-induced currents but no set-up):

$$\frac{dS_{xx}}{dx} + \rho g H\,\frac{d\bar{\eta}}{dx} = 0 \tag{B38}$$

where $H = d + \bar{\eta}$ is the total water depth (including the wave-induced set-up), *d* is the bottom level, $\bar{\eta}$ is the mean surface elevation (including the wave-induced set-up) and

$$S_{xx} = \rho\,g\int\left[n\cos^2\theta + n - 1/2\right]E\,d\sigma\,d\theta$$

is the radiation stress tensor.

Observation and computations based on the vertically integrated momentum balance equation of Dingemans et al. (1987) show that the wave-induced currents are mainly driven by the divergence-free part of the wave forces whereas the set-up is mainly due to the rotation-free part of these forces. To compute the set-up, it would then be sufficient to consider the divergence of the momentum balance equation. If the divergence of the acceleration in the resulting equation is ignored, the result is:

$$\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial}{\partial x}\left(\rho g H\,\frac{\partial\bar{\eta}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho g H\,\frac{\partial\bar{\eta}}{\partial y}\right) = 0 \tag{B39}$$

Diffraction

In a simplest case, we assume there are no currents. This means that c_σ=0. Let denotes the propagation velocities in geographic and spectral spaces for the situation without diffraction as: $c_{x,0}$,0,c_{y0} and $c_{\theta 0}$. These are given by:

$$c_{x,0} = \frac{\partial \omega}{\partial k} \cos(\theta) \ , \quad c_{y,0} = \frac{\partial \omega}{\partial k} \sin(\theta) \ , \quad c_{\theta,0} = -\frac{1}{k} \frac{\partial \omega}{\partial h} \frac{\partial h}{\partial n} \tag{B40}$$

where k is the wave number and n is perpendicular to the wave ray. We consider the following eikonal equation:

$$K^2 = k^2(1+\delta) \tag{B41}$$

with δ denoting the diffraction parameter as given by:

$$\delta = \frac{\nabla(cc_g \nabla H_s)}{cc_g H_s} \tag{B42}$$

Due to diffraction, the propagation velocities are given by:

$$c_x = c_{x,0}\overline{\delta} \ , \quad c_y = c_{y,0}\overline{\delta} \ , \quad c_\theta = c_{\theta,0}\overline{\delta} - \frac{\partial \overline{\delta}}{\partial x} c_{y,0} + \frac{\partial \overline{\delta}}{\partial y} c_{x,0} \tag{B43}$$

where $\overline{\delta}$ $\sqrt{1+\delta}$. =

Numerical implementation

The integration of the action balance equation has been implemented in SWAN with finite difference schemes in all five dimensions (time, geographic space and spectral space). These are first described for the propagation of the waves without the source terms for generation, dissipation and wave-wave interactions. Then the implementation of these source terms is described.

Time is discretized with a simple constant time step Δt for the simultaneous integration of the propagation and the source terms. This is different from the time discretization in the WAM model or the WAVEWATCH III model where the time step for propagation is different from the time step for the source terms. Geographic space is discretized with a rectangular grid with constant resolutions Δx and Δy in x and y-direction respectively (in fact, this rectangular grid is a special case of the curvi-linear grid that has been programmed in SWAN. The spectrum in the model is discretized with a constant directional resolution $\Delta\theta$ and a constant relative frequency resolution $\Delta \sigma / \sigma$ (logarithmic frequency distribution). For reasons of economy, an option is available to compute only wave components travelling in a pre-defined directional sector ($\theta_{min} < \theta < \theta_{max}$; e.g., those components that travel shorewards within a limited directional sector). The discrete frequencies are defined between a fixed low-frequency cut-off and a fixed high-frequency cut-off (the prognostic part of the spectrum). For these frequencies the spectral density is unconstrained. Below the low-frequency cut-off (typically f_{min} =0.04 Hz for field conditions) the spectral densities are assumed to be zero. Above the fixed high-frequency cut-off (typically 1 Hz for field conditions) a diagnostic f^m tail is added. This tail is used to compute nonlinear wave-wave interactions at the high frequencies and to compute integral wave parameters. The reason for using a fixed high-frequency cut-off rather than a dynamic cut-off frequency that depends on the wind speed or on the mean frequency, as in WAM and WAVEWATCH III, is that in coastal regions mixed sea states with rather different characteristic frequencies may occur. For instance, a local wind may generate a very young sea behind an island, totally unrelated to (but superimposed on) a simultaneously occurring swell. In such cases a dynamic cut-off frequency may be too low to properly account for the locally generated sea state. Based on physical arguments the value of m (the power in the above expression of the spectral tail) should be between 4 and 5 (e.g., Phillips, 1985). In SWAN m=4 if the wind input formulation of Komen et al. (1984) is used (cf. WAM Cycle 3), and m=5 if the wind input formulation of Janssen (1991a) is used (cf. WAM Cycle 4).

Propagation The numerical schemes in SWAN have been chosen on the basis of robustness, accuracy and economy. Since the nature of the basic equation is such that the state in a grid point is determined by the state in the upwave grid points (so-called hyperbolic property), the most robust and efficient scheme is an implicit upwind scheme. For such a scheme, the values of the time and space steps Δt, Δx *and* Δy are mutually independent. An implicit scheme may also be economical in the sense that such a scheme is unconditionally stable. It permits relatively large time steps in the computations (much larger than for explicit schemes in shallow water). Several years of experience in using the second-generation HISWA shallow water wave model (Holthuijsen et al., 1989) has shown that for coastal regions a first-order upwind difference scheme in geographic space is usually accurate enough. This experience, together with test computations with SWAN has also shown that in spectral space a higher accuracy than that of a first-order upwind scheme is required. This can be achieved by supplementing such a scheme with a second-order central approximation (more economic than a second-order upwind scheme). For SWAN therefore, implicit upwind schemes in both geographic and spectral space have been chosen, supplemented with a central approximation in spectral space.

The fact that in geographic space, the state in a grid point is determined by the state in the upwave grid points (as defined by the direction of propagation), permits a decomposition of the spectral space into four quadrants. In each of the quadrants the computations can be carried out independently from the other quadrants except for the interactions between them due to refraction and nonlinear wave-wave interactions. The wave components in SWAN are correspondingly propagated in geographic space with an upwind scheme (upwind is the common term in numerical analysis, but up-wave would be more appropriate in the case of SWAN). SWAN contains three such schemes:

3. First-order backward space, backward time (BSBT) scheme. It is in fact a classical first-order upwind scheme. To be used for both stationary and non-stationary small-scale applications.
4. Second-order upwind scheme with second-order diffusion. This scheme is called the SORDUP scheme, although it is known as the BDF scheme in the CFD community. To be used for stationary large-scale applications.
5. Second-order upwind scheme with third-order diffusion as proposed in Stelling and Leedertse (1992). This so-called S&L scheme can be used for non-stationary large-scale applications.

The first-order BSBT scheme (not default in SWAN) will be discussed first and then the extension to the higher order schemes which are default in SWAN. The BSBT scheme is a sequence of four forward-marching sweeps (one per quadrant). To properly account for the energy transfer between the four quadrants, the computations are carried out iteratively within each time step. The integration in time is a simple backward finite difference, so that the discretization of the action balance equation is (for positive propagation speeds c_x, c_y >0 , including the computation of the source terms but ignoring their discretization):

$$\left[\frac{N^{i_t,n}-N^{i_t-1}}{\Delta t}\right]_{i_x,i_y,i_\sigma,i_\theta}+\left[\frac{[c_xN]_{i_x}-[c_xN]_{i_x-1}}{\Delta x}\right]^{i_t,n}_{i_y,i_\sigma,i_\theta}+\left[\frac{[c_yN]_{i_y}-[c_yN]_{i_y-1}}{\Delta y}\right]^{i_t,n}_{i_x,i_\sigma,i_\theta}+$$
$$\left[\frac{(1-\nu)[c_\sigma N]_{i_\sigma+1}+2\nu[c_\sigma N]_{i_\sigma}-(1+\nu)[c_\sigma N]_{i_\sigma-1}}{2\Delta\sigma}\right]^{i_t,n}_{i_x,i_y,i_\theta}+ \quad \text{(C1)}$$
$$\left[\frac{(1-\eta)[c_\theta N]_{i_\theta+1}+2\eta[c_\theta N]_{i_\theta}-(1+\eta)[c_\theta N]_{i_\theta-1}}{2\Delta\theta}\right]^{i_t,n}_{i_x,i_y,i_\sigma}=\left[\frac{S}{\sigma}\right]^{i_t,n^*}_{i_x,i_y,i_\sigma,i_\theta}$$

where i_t is the time-level index and i_x ,i_y ,i_σ and i_0 are grid counters and Δt, Δx, Δy, $\Delta\sigma$ and $\Delta\theta$ are the increments in time, geographic space and spectral space, respectively. The iterative nature of the computation is indicated with the iteration index n (the iteration index for the source terms n^* is equal to n or n-1, depending on the linearization of the source term, see below). For negative propagation speeds, appropriate + and -signs are required in Eq. (C1).

The coefficients υ and η determine the degree to which the scheme in spectral space is upwind or central. They thus control the numerical diffusion in frequency and directional space, respectively. A value of υ=0 or η=0 corresponds to central schemes which have the largest accuracy (numerical diffusion.0). Value of υ=1 or η=1 correspond to upwind schemes which are somewhat more diffusive and therefore less accurate but more robust. If large gradients of the action density in frequency space or directional space are present, numerical oscillations can arise (especially with the central difference schemes) resulting in negative values

of the action density. In each sweep such negative values are removed from the two-dimensional spectrum by setting these values equal to zero and rescaling the remaining positive values such that the frequency-integrated action density per spectral direction is conserved. The depth derivatives and current derivatives in the expressions of c_σ and c_0 are calculated with a first-order upwind scheme. For very strong refraction the value of c_0 is reduced in each grid point and for each wave component individually with the square of the fraction of the grid spacing over which kd<3.0.

For stationary conditions, SWAN can be run in stationary mode. Time is then removed as a variable but the integration (in geographic space) is still carried out iteratively. The propagation scheme is still implicit as the derivatives of action density (in x or y) at the computational level (i_x or i_y, respectively) are formulated at that level except in the integration dimension (x or y; depending on the direction of propagation) where also the upwave level is used. The values of Δx and Δy are therefore still mutually independent.

For the SORDUP scheme, which is the default scheme for stationary computations, the two terms in equation (C1) representing x-and y-derivatives are replaced by (Rogers *et al.* (2002)):

$$\left[\frac{1.5\,[c_xN]_{i_x} - 2\,[c_xN]_{i_x-1} + 0.5\,[c_xN]_{i_x-2}}{\Delta x}\right]^{i_t,\,n}_{i_y,\,i_\sigma,\,i_\theta} + \left[\frac{1.5\,[c_yN]_{i_y} - 2\,[c_yN]_{i_y-1} + 0.5\,[c_yN]_{i_y-2}}{\Delta y}\right]^{i_t,\,n}_{i_x,\,i_\sigma,\,i_\theta} \qquad \text{(C1a)}$$

For the S&L scheme, which is the default scheme for non-stationary computations, the two terms in equation (C1) representing x-and y-derivatives are replaced by (Rogers *et al.* (2002)):

$$\left[\frac{\frac{5}{6}\,[c_xN]_{i_x} - \frac{5}{4}\,[c_xN]_{i_x-1} + \frac{1}{2}\,[c_xN]_{i_x-2} - \frac{1}{12}\,[c_xN]_{i_x-3}}{\Delta x}\right]^{i_t,\,n}_{i_y,\,i_\sigma,\,i_\theta} + \left[\frac{\frac{5}{6}\,[c_yN]_{i_y} - \frac{5}{4}\,[c_yN]_{i_y-1} + \frac{1}{2}\,[c_yN]_{i_y-2} - \frac{1}{12}\,[c_yN]_{i_y-3}}{\Delta y}\right]^{i_t,\,n}_{i_x,\,i_\sigma,\,i_\theta} \qquad \text{(C1a)}$$

$$+ \left[\frac{\frac{1}{4}\,[c_xN]_{i_x+1} - \frac{1}{4}\,[c_xN]_{i_x-1}}{\Delta x}\right]^{i_t-1}_{i_y,\,i_\sigma,\,i_\theta} + \left[\frac{\frac{1}{4}\,[c_yN]_{i_y+1} - \frac{1}{4}\,[c_yN]_{i_y-1}}{\Delta y}\right]^{i_t-1}_{i_x,\,i_\sigma,\,i_\theta} \qquad \text{(C1b)}$$

In the neighbourhood of open boundaries, land boundaries and obstacles (i.e., the last two grids adjoining such grid points for the SORDUP scheme and the last three grids adjoining such grid points for the S&L scheme), SWAN will revert to the first-order BSBT scheme. This scheme has a larger numerical diffusion but that is usually acceptable over the small distances involved.

Usually, the numerical diffusion of the S&L scheme is so small that the so-called garden-sprinkler effect (GSE) may show up if propagation over very large distances is considered. This effect is due to the spectral resolution (see Booij and Holthuijsen (1987)). It can be counteracted by a diffusion term that has been explicitly added to the numerical scheme (not default in SWAN). Its value depends on the spectral resolution and the propagation time of the waves (see the input variable [wave age] in the PROP command).

The diffusion applied in the propagation direction is:

$$D_{ss} = \Delta c^2 T/12 \tag{C2}$$

where T is the wave age.

The diffusion normal to the propagation direction is:

$$D_{ss} = \Delta c^2 T/12 \tag{C3}$$

From these, diffusion coefficients are calculated as:

$$\begin{aligned} D_{xx} &= D_{ss}\cos^2\theta + D_{nn}\sin^2\theta, \\ D_{yy} &= D_{ss}\sin^2\theta + D_{nn}\cos^2\theta \text{ and} \\ D_{xy} &= (D_{ss} - D_{nn})\cos\theta\sin\theta. \end{aligned} \tag{C4}$$

The diffusion terms are computed at the time level i_t -1. The diffusion terms are computed as follows:

$$D_{xx}\left[\frac{[N]_{i_x+1} - 2[N]_{i_x} + [N]_{i_x-1}}{\Delta x^2}\right]^{i_t-1}_{i_y,\, i_\sigma,\, i_\theta}$$

$$D_{yy}\left[\frac{[N]_{i_y+1} - 2[N]_{i_y} + [N]_{i_y-1}}{\Delta y^2}\right]^{i_t-1}_{i_x,\, i_\sigma,\, i_\theta} \tag{C5}$$

$$D_{xy}\left[\frac{[N]_{i_x,i_y} - [N]_{i_x-1,i_y} - [N]_{i_x,i_y-1} + [N]_{i_x-1,i_y-1}}{\Delta x \Delta y}\right]^{i_t-1}_{i_\sigma,\, i_\theta}$$

This explicit finite differencing is fast (having little impact on computation time) but only conditionally stable. Through mathematical analysis (not shown) it can be shown that a likely stability condition for the one-dimensional S&L scheme with this GSE correction is $D\Delta t / (\Delta x^2) \leq 0.5$ which corresponds to the two-dimensional stability criterion of Tolman (1995; based on Fletcher, 1988, Part I, section 7.1.1):

$$Q = \frac{\max(D_{xx}, D_{yy}, D_{xy})\,\Delta t}{\min(\Delta x, \Delta y)^2} \leq 0.5 \tag{C6}$$

Thus, it is credible that Eq. (C6) holds true for the two-dimensional S&L scheme with this GSE correction. In experiments, it was found that with $Q \leq 0.48$, no instability was observed. In short, by adding the GSE correction, the unconditionally stable advection scheme of SWAN becomes a (likely) conditionally stable advection-diffusion scheme. It is readily shown that for typical ocean applications D_{nn} dominates the diffusion and Q can be written as:

$$Q = \overline{C}\,T/\Delta x \,.\, \overline{C}\Delta t/\Delta x \,.\, \Delta\theta^2/12 \tag{C7}$$

The variable wave age *T* could be computed during the computations of SWAN (Booij and Holthuijsen, 1987) but it requires the same order of magnitude of computer memory as integrating the action balance equation. Instead a constant wave age *T* can be used as an approximation, so that Eq.(C7) becomes

$$Q = \overline{L} / \Delta x \,.\, \mu \,.\, \Delta\theta^2 / 12 \qquad \text{(C8)}$$

where the characteristic travel distance of the waves is $\overline{L} = \overline{c}\ \overline{T}$ (e.g., the dimension of the ocean basin). For oceanic applications, the Courant number is typically $\mu = 1/2$ so that $Q \leq 0.25$ for typical values of $\Delta\theta$ and $\overline{L} / \Delta x$ (the number of grid point in one direction of the grid). This implies that the S&L scheme with this GSE correction is stable for typical ocean cases. For shelf sea (regional) applications, the value of $\mu = O(1)$ but the garden-sprinkler effect tends to be small on these scales and the diffusion can and should not be used to avoid the stability problem. For small-scale (local) applications, typically $\mu = O(10\text{-}100)$. But such cases are usually treated as stationary and the SORDUP scheme should be used (no GSE correction is included in this scheme).

Curvi-linear grid

The propagation scheme in SWAN for geographic space is formulated on a curvi-linear geographic grid (irregular, quadrangular, not necessarily orthogonal) rather than the recti-linear grid of SWAN Cycle 1. This modification is based on approximating the geographic distribution of the energy (action) density between each three neighbouring grid points with a flat triangle. The gradient in each grid point at location (xi,yj) is then readily approximated from the up-wind grid points. For the x-direction, this is for grid point *i*,*j* (the grid points are ordered in *x*,*y*-space with labels *i* and *j* respectively):

$$\frac{\partial c_x N}{\partial x} \approx \left[\frac{[c_x N]_{i,j} - [c_x N]_{i-1,j}}{\Delta \tilde{x}_1} \right] + \left[\frac{[c_x N]_{i,j} - [c_x N]_{i,j-1}}{\Delta \tilde{x}_2} \right] \qquad \text{(C9)}$$

where $\Delta \tilde{x}_1 = \Delta x_1 - (\Delta y_1 / \Delta y_2)\, \Delta x_2$, $\Delta \tilde{x}_2 = \Delta x_2 - (\Delta y_2 / \Delta y_1)\, \Delta x_1$. The increments are $\Delta x_1 = x_{i,j} - x_{i-1,j}$, $\Delta x_2 = x_{i,j} - x_{i,j-1}$, $\Delta y_1 = y_{i,j} - y_{i-1,j}$ and $\Delta y_2 = y_{i,j} - y_{i,j-1}$.

The gradient in y-direction is similarly approximated.

Boundary conditions

The boundary conditions in SWAN, both in geographic space and spectral space are fully absorbing for wave energy that is leaving the computational domain or crossing a coast line. The incoming wave energy along open geographic boundaries, needs to be prescribed by the user. For coastal regions such incoming energy is usually provided only along the deep-water boundary and not along the lateral geographic boundaries (i.e., the spectral densities are assumed to be zero). This implies that such erroneous lateral boundary conditions may be propagated into the computational area. The affected areas are typically triangular regions with the apex at the corners between the deep-water boundary and the lateral boundaries, spreading towards shore at an angle of 30^{o} to 45^{o} (for wind sea conditions) on either side of the deep-water mean wave direction (less for swell conditions; this angle is essentially equal to the one-sided width of the directional distribution of the incoming wave spectrum). For this reason the lateral boundaries should be sufficiently far away from the area of interest to avoid the propagation of this error into the area.

Solution technique

To explain the numerical solution technique for solving equation (C1), first, ignore the decomposition in quadrants. The propagation of the waves in both geographic and spectral space would then be described with one large basic matrix which can be solved in several ways. Removing refraction, frequency shifting and nonlinear source terms from this basic matrix permits a matrix solution with a Gauss-Seidel technique in which the matrix is decomposed in four sections (the above four directional quadrants) which are each solved in one step (super-convergence), see, e.g., Golub and Van Loan (1986). Restoring refraction and frequency shifting to the matrix requires the solution of a submatrix for each geographic grid point. If no currents are present and the depth is stationary, i.e. $c_\sigma = 0$, a tridiagonal submatrix results that can be solved easily with the Thomas' algorithm (see Golub and Van Loan (1986)). If currents are present or the depth is non-stationary, a penta-diagonal submatrix for each geographic point occur. It is solved with the fast iterative S(trongly) I(mplicit) P(rocedure) solver of Stone (1968). Because of refraction and frequency-shifting, equation (C1) needs to be solved iteratively until some break-off criteria are met. To reduce the number of iterations in stationary mode with wind generation, SWAN starts with a reasonable first-guess of the wave field. This first-guess is based on the second-generation source terms of Holthuijsen and De Boer (1988),

adapted for shallow water. It reduces the number of iterations typically by a factor two. In non-stationary mode, a very reasonable first-guess per time step is available from the previous time step and the number of iterations is expected to be small.

Generation, wave-wave interactions and dissipation

The numerical approximation of the source terms in SWAN is essentially implicit.

The linear growth term *A* is independent of integral wave parameters and the energy density and can therefore be readily computed. All other source terms depend on energy density and they can be described as a (quasi-)linear term: $S = \varphi E$, in which φ is a coefficient that depends on (integral) wave parameters (e.g., E_{tot}, $\overline{\sigma}$, $\tilde{k}$, σ, k, etc.) and action densities. Since these are only known at the previous iteration level n-1, the coefficient φ need to be determined at that iteration level.

For positive source terms (wind input and the triad wave-wave interactions if positive) the integration is generally more stable if an explicit formulation is used (i.e., the source term depends on E^{n-1} and not on E^n) rather than an implicit formulation (i.e., the source term depends also on E^n). The explicit formulation for these source terms in SWAN is therefore:

$$S^n \approx \varphi^{n-1} E^{n-1} \tag{C10}$$

For reasons of economy this explicit approximation is also used for the formulation of the quadruplet wave-wave interactions (for both the positive and negative contributions). This is considered reasonable since Tolman (1992a) has shown that using an explicit formulation in combination with a limiter (see below) gives similar results as the use of a more expensive implicit scheme (this implicit formulation is also optionally available in SWAN; in the WAM model it is indicated as the semi-implicit scheme, the WAMDI group, 1988; Komen et al, 1994).

For negative source terms the integration is generally more stable if an implicit scheme is used. The strongly nonlinear, negative source term of depth-induced wave breaking at iteration level n is accordingly estimated with a linear approximation:

$$S^n \simeq \Phi^{n-1} E^{n-1} + \left(\frac{\partial S}{\partial E} \right)^{n-1} (E^n - E^{n-1}) \quad . \tag{C11}$$

However, to achieve even more stable computations for this source term, the term $\varphi^{n-1} E^{n-1}$ in this formulation has been replaced by $\varphi^{n-1} E^n$ (making the formulation somewhat more implicit and thus more robust; note that in the limit the solution is the same). Since this process of depth-induced wave breaking has been formulated such that $S = a S_{tot}$ and $E = a E_{tot}$ the derivative $\partial S / \partial E$ is analytically determined as $\partial S_{tot} / \partial E_{tot}$ (where *a* is identical in both expressions and the total energy E_{tot} and the total source S_{tot} are the integrals over all frequencies and directions of E (σ, θ) and $S_{ds,br}$ (σ, θ), respectively). For the other negative (mildly nonlinear) source terms, i.e., whitecapping, bottom friction and negative triad wave-wave interactions a similar accuracy of estimating S^n can be achieved with the following simpler, and therefore more economical approximation in which $(\partial S / \partial E)^{n-1}$ of Eq. (C11) has been replaced by $(S/E)^{n-1}$

$$S^n \simeq \Phi^{n-1} E^{n-1} + \left(\frac{S}{E} \right)^{n-1} (E^n - E^{n-1}) \quad . \tag{C12}$$

With $S = \varphi E$, this reduces to:

$$S^n \simeq \Phi^{n-1} E^n \quad . \tag{C13}$$

These approximations for the source terms are added to the elements of the matrix for the propagation.

Action limiting and frequency-dependent under-relaxation

Many time scales are involved in the evolution of wind waves. The high-frequency waves have much shorter

time scales than the low-frequency waves. To deal with these large differences in time scales, efficient numerical integration techniques need to be designed for operational applications.

To match physical time scales at relatively high frequencies, the maximum total change of action density per iteration at each discrete wave component is limited to a fraction of 10% of the Phillips (1957) equilibrium level (reformulated in terms of action density and wave number to be applicable in shallow water; as in the WAM model and in the WAVEWATCH III model; Tolman, 1992a):

$$\left| \Delta N(\sigma,\theta) \right|_{\max} = \frac{0.1}{2\pi\sigma} \frac{\alpha_{PM}\pi}{k^3 c_g} \qquad \text{(C14)}$$

where α_{PM} = 0.0081 is the Phillips' constant of the Pierson-Moskowitz (1964) spectrum. This approach appears to guarantee numerical stability at relatively large time steps even when they violate the time scales of wave growth. Since, low-frequency waves carry the most energy it is necessary to solve the action density equation for slowly varying waves accurately without dictating them by a limiter, whereas for high-frequency waves only equilibrium solution suffice. However, numerous studies with the stationary mode of SWAN show that limiter activity is not restricted to the equilibrium range, but that it is also found at the lower-frequency, energy-rich part of the spectrum. Moreover, action limiting often leads to small oscillations in the converged solution.

An alternative route is under-relaxation, i.e. to make smaller updates by means of a much smaller time step. However, this greatly increases the computational time. An approach, that finds a compromise between fast convergence and minimizing the role of the limiter for particularly low frequencies, is a frequency-dependent under-relaxation technique as proposed by M. Zijlema. The basic idea is to link the extent of updates to the wave frequency !the larger the frequency the smaller the updates.

The discretization of the action density equation at each geographical gridpoint yields a system of equations which can be written as

$$AN = b \qquad \text{(C15)}$$

where *A* and *b* represent space discretizations and source terms, respectively, and *N* denotes an algebraic vector containing the action density in spectral bins. Carrying out smaller updates may be achieved by enhancing both the main diagonal of *A* and *b* with certain amounts. To make them frequency-dependent, we proposed the following:

$$A \leftarrow A + \alpha\sigma I \; ; \qquad b \leftarrow b + \alpha\sigma N^{n-1} \qquad \text{(C16)}$$

where α is a proportionality factor. Note that the added amounts will not affect the solution of the original system of equations when steady-state is reached. It is found that the method of under-relaxation yields more stable model results than action limiting, albeit by using more computational time.
Further details on action limiting and frequency-dependent under-relaxation can be found in Zijlema and Van der Westhuysen (2004).

Wave-induced set-up

In 1D cases, the wave-induced set-up is calculated in SWAN with a simple trapezoidal rule.

In 2D cases, the Poisson equation of the divergence-free force field is solved in SWAN with a modified Successive Over Relaxation (SOR) technique (Botta and Ellenbroek, 1985). The boundary conditions for this elliptical partial differential equation are:

a1) at open boundaries: equilibrium between wave force and hydrostatic pressure gradient normal to the model boundary,
a2) at last grid points before shoreline: equilibrium between wave force and hydrostatic pressure gradient normal to the model boundary and
a3) at deepest boundary point: set-up is zero.

The shoreline in SWAN moves as dictated by the wave-induced set-up. The set-up computations are available in both the recti-linear and curvi-linear grids. It is however not supported in parallel runs using MPI and also not tested with OpenMP.